2022

Inflation

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CONTENTS

01

Motivation

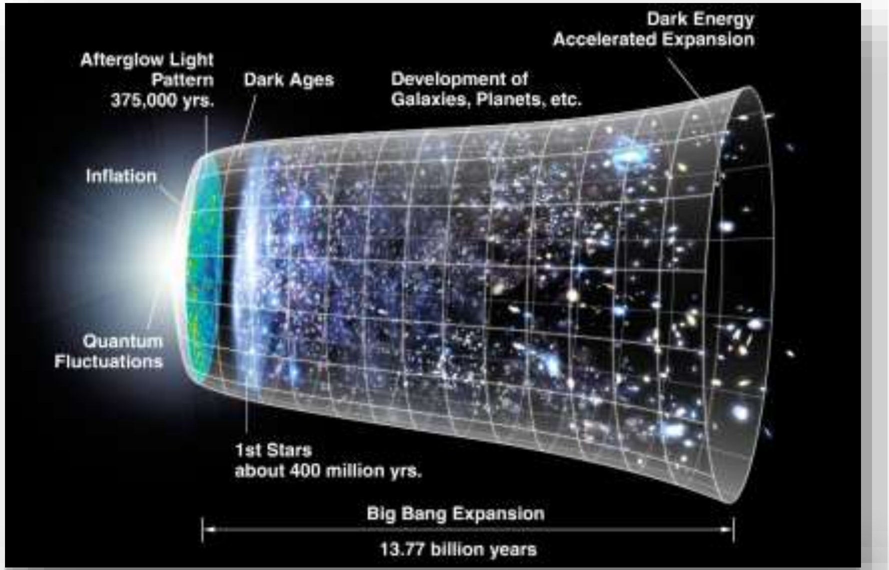
Motivation

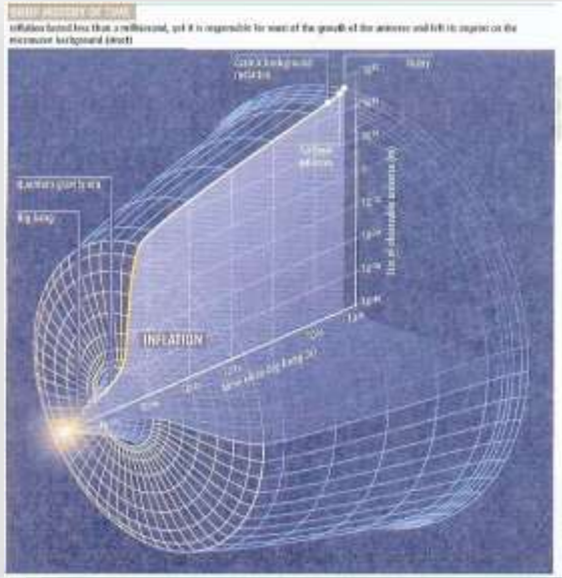
Flatness Problem

&

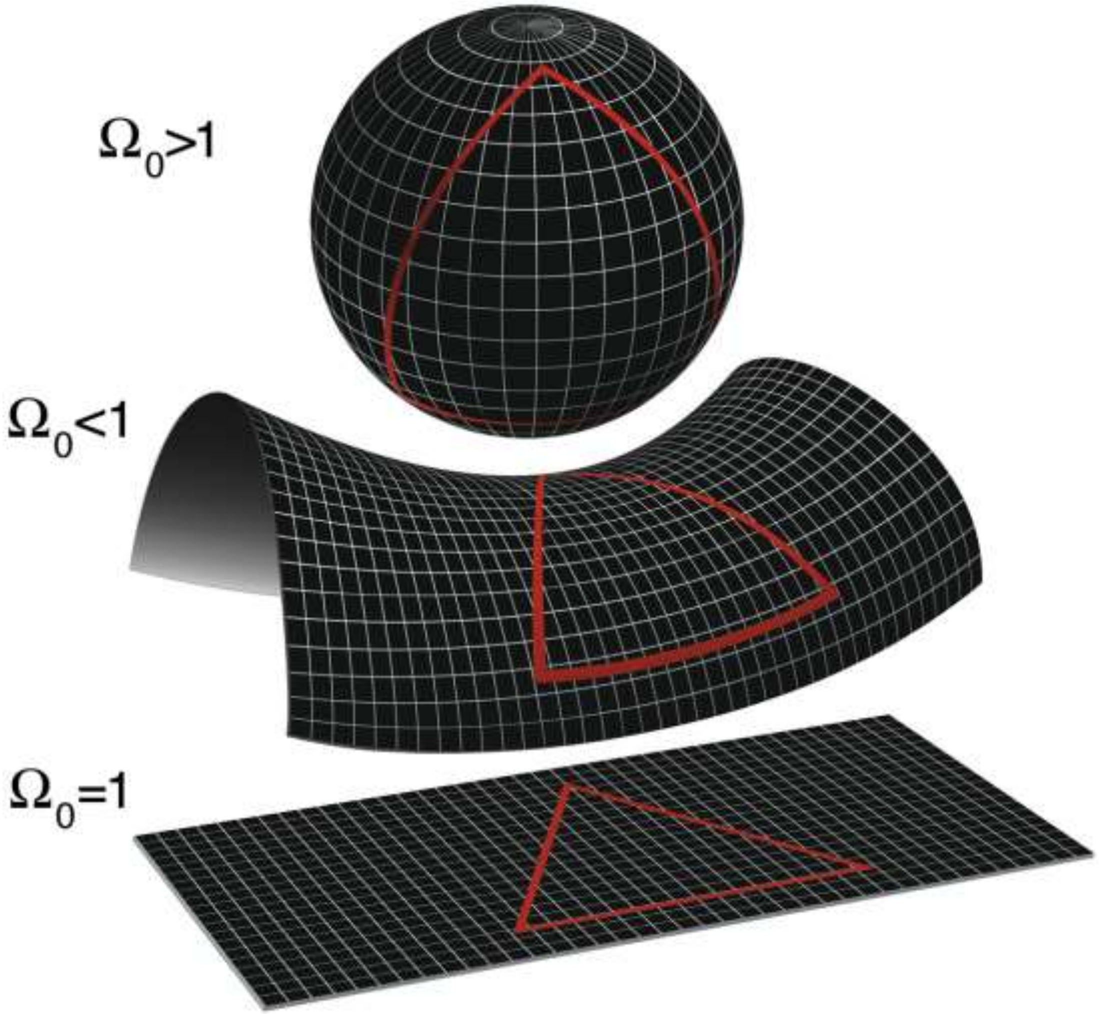
Horizon Problem

Inflation is a scenario to address this question, at least to some extent. Inflation is a period in the very early universe, when the expansion of the universe was accelerating.





Flatness Problem



The Friedmann equation:

$$\Omega - 1 = \frac{K}{a^2 H^2}$$

$$\Omega_k = 1 - \Omega$$

CMB:

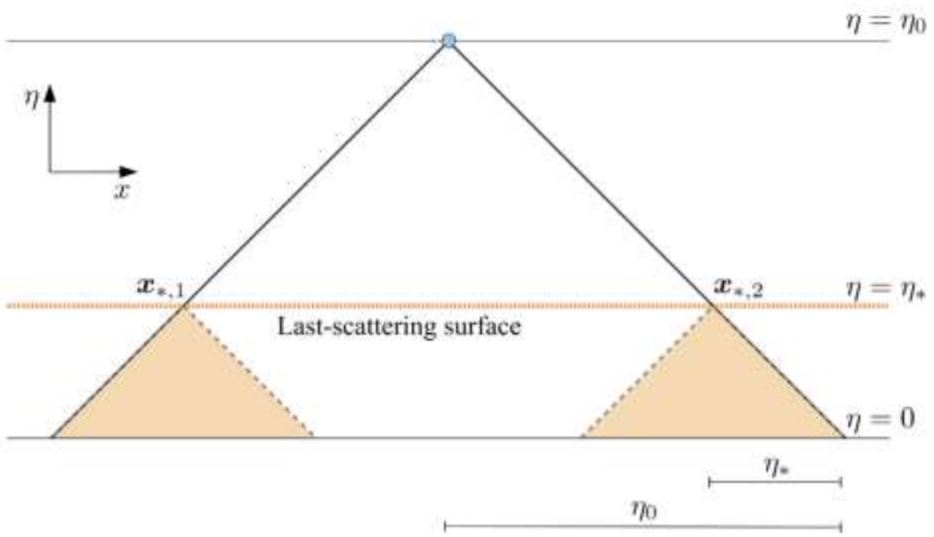
$$|\Omega_k| = 0.030^{+0.026}_{-0.025} \ll 1$$

Then:

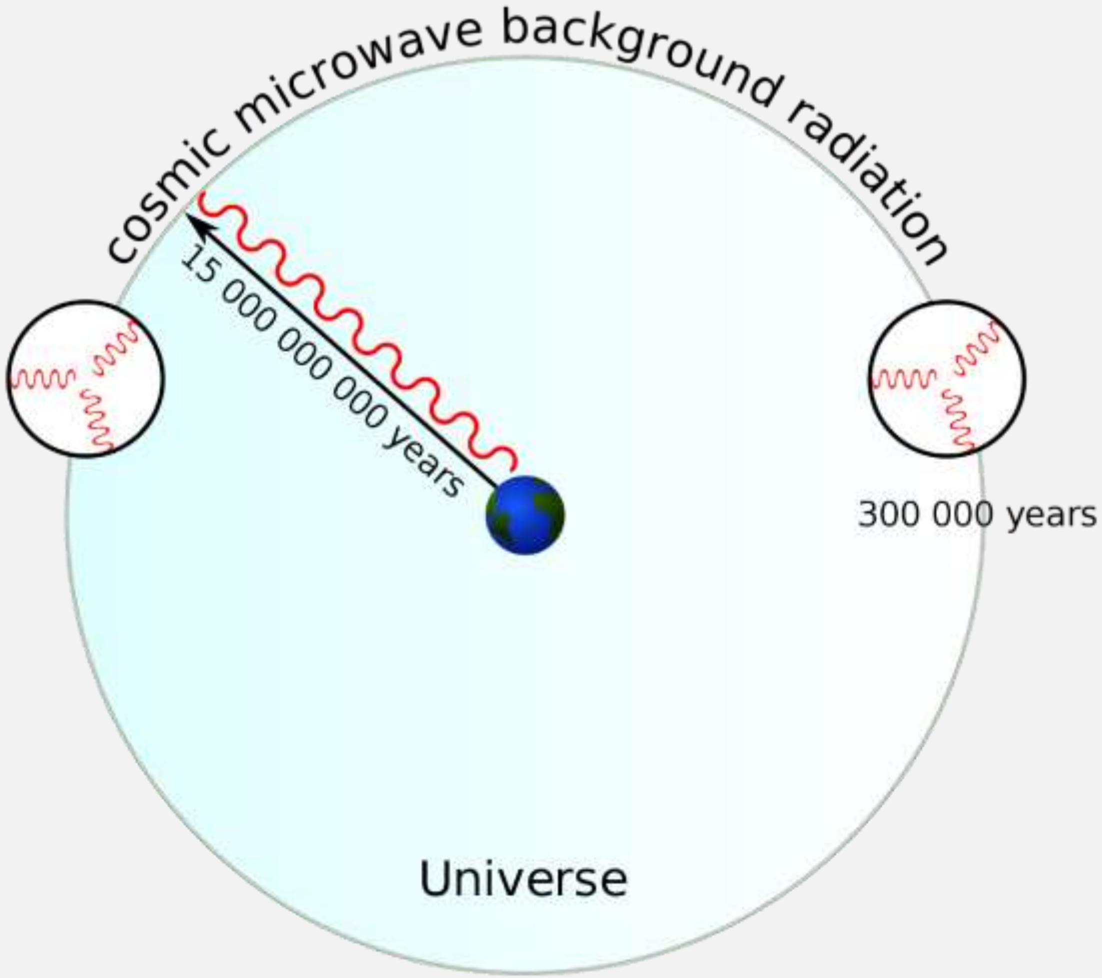
$$|\Omega_k| \ll 10^{-16}$$

What a COINCIDENCE!

Horizon Problem



Regions on the CMB sky separated by more than about 1° had not had time to interact, yet their temperature is the same with an accuracy of 10^{-4}.



02

Inflation: The Solution

Inflation: The Solution to Flatness Problem

The origin of the flatness problem is that $|\Omega - 1| = \frac{|K|}{(aH)^2}$ grows with time. Now:

$$\frac{d}{dt}|\Omega - 1| = |K|\frac{d}{dt}\left(\frac{a}{a^2H^2}\right) = |K|\frac{d}{dt}\left(\frac{1}{\dot{a}^2}\right) = \frac{-2|K|}{\dot{a}^3}\ddot{a}$$

The Problem is from

$$\ddot{a} < 0$$

Thus the reason for the flatness problem is that the expansion of the universe is

Decelerating.

So an epoch of Acceleration will solve the problem.

Inflation: The Solution to Horizon Problem

the comoving horizon η is the logarithmic integral of the comoving Hubble radius:

$$\eta(a) = \int_0^a \mathrm{d}\, ln\, a' \frac{1}{a' H(a')}$$

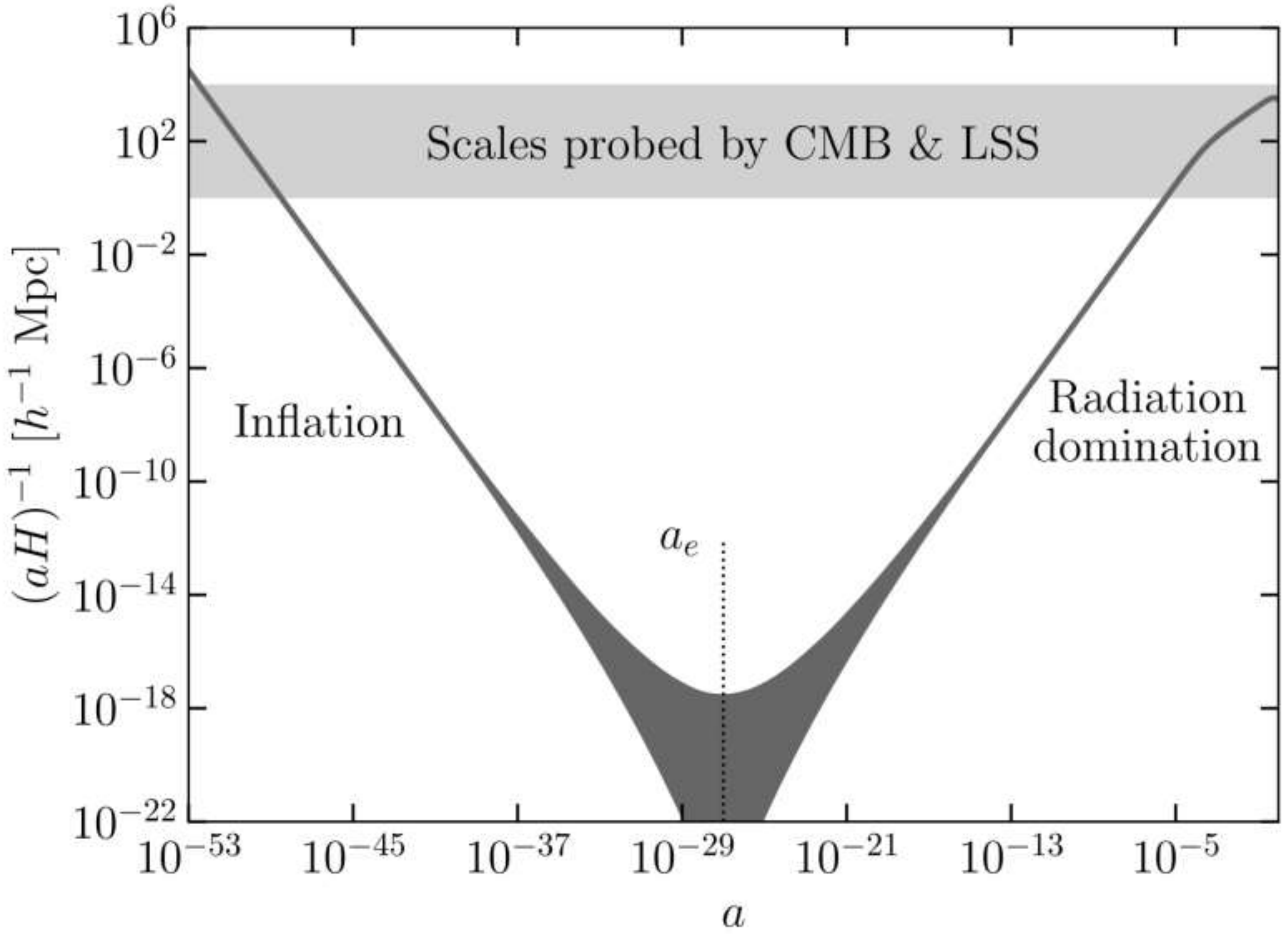
The comoving Hubble radius $\frac{1}{aH(a)}$ is always Increasing!

This points the way to a solution:

If there was an early epoch during which the comoving Hubble radius Decreased?

i.e. $\ddot{a} > 0$

So, an epoch of early acceleration would solve the horizon problem. This postulated epoch is called inflation.



Scales probed by CMB & LSS
Inflation
Radiation domination
a_e
$(aH)^{-1}$ [h^{-1} Mpc]
a
10^{6}
10^{2}
10^{-2}
10^{-6}
10^{-10}
10^{-14}
10^{-18}
10^{-22}
10^{-53}
10^{-45}
10^{-37}
10^{-29}
10^{-21}
10^{-13}
10^{-5}

Conclusion

- Flatness problem solved: The flatness problem is solved, since during inflation $|\Omega - 1|$ is shrinking.

- Horizon Problem solved: The horizon problem is solved, since during inflation the causally connected region is shrinking.

03

Quintessence Field

Quintessence

The simplest way to generate such a transitory epoch of accelerated expansion is via the potential energy of a scalar field, the homogeneous one is called Quintessence.

We propose a scaler field ϕ, then we get the energy-momentum tensor for it:

$$T^{\alpha}_{\beta} = g^{\alpha\nu}\frac{\partial\phi}{\partial x^{\nu}}\frac{\partial\phi}{\partial x^{\beta}} - \delta^{\alpha}_{\beta}\left[\frac{1}{2}g^{\mu\nu}\frac{\partial\phi}{\partial x^{\mu}}\frac{\partial\phi}{\partial x^{\nu}} + V(\phi)\right]$$

Kick out the space derivatives(homogeneous), we get:

$$T^{\alpha}_{\beta} = -\delta^{\alpha}_{0}\delta^{0}_{\beta}\dot{\phi}^{2} + \delta^{\alpha}_{\beta}\left[\frac{1}{2}\dot{\phi}^{2} - V(\phi)\right]$$

Quintessence

Fit it in ideal fluid, we get:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

This is equivalently phrased as an equation of state:

$$w = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

that is close to -1.

Quintessence

With Friedman equation:

$$\dot{\rho} = -3(\rho + P)\frac{\dot{a}}{a}$$

The evolution of ϕ for any potential can be derived:

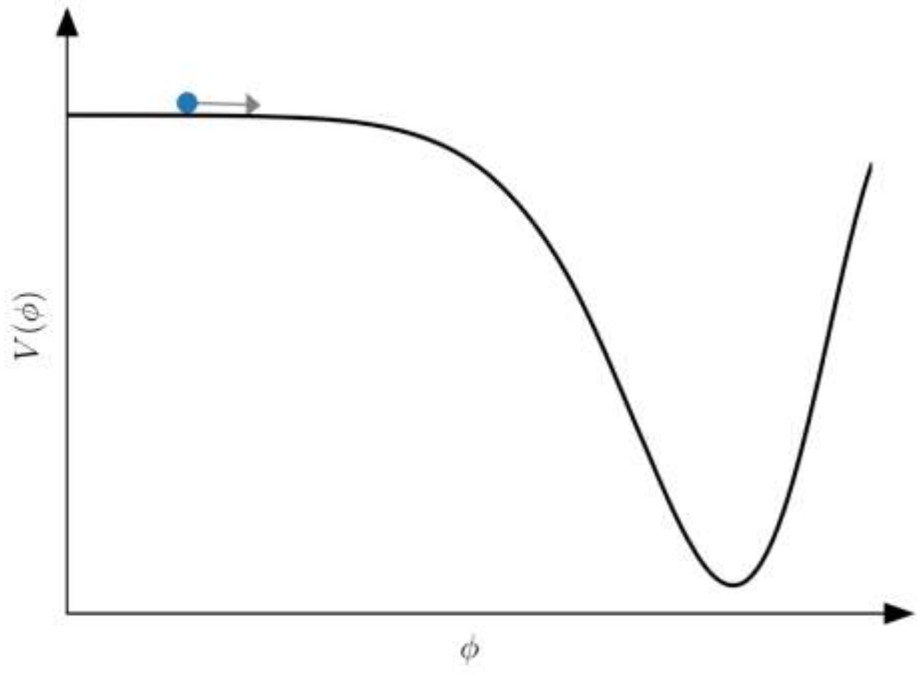
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$

Above is called "Klein-Gordon Equation".

04

Slow-roll Inflation

Slow-roll Inflation



A model of inflation consists of:

1. a potential $V(\phi)$.
2. a way of ending inflation.

The potential energy of such a field is very close to constant, so it quickly comes to dominate over the kinetic energy (and the energy of all other particles), inflation ends once the field has reached the minimum of the potential.

05

Reference

Reference

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THANKS!

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